A Systemic Application of Graph Theory in Issues of Public Administration

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Abstract
The Graph Theory is a branch of Discrete Mathematics that is specialized in describing various phenomena through the depiction of entities and relationships among them. The application of graphs may enhance the study of various issues of public administration in combination with the systemic methodology. This concept is exemplified in this article through the presentation of two issues of public administration’s interest, namely: the chain of command and the taxation policy.

Keywords: systemic theory; public administration; graph theory; taxation policy; chain of command.

1. Introduction
The contribution of Mathematics, and their significance, in Public Administration is both fundamental and obvious. The most utilized branches of Applied Mathematics, for the purpose of assisting decision making and management in public organizations are Statistics and Economics (Mathematics of Accounting, especially). Their impact is well documented (Grima, 2011; Artal & Sales, 2011). Another branch, which is not published, is Cryptography (Gómez, 2011). Yet, there is also a branch of Discrete Mathematics always present, namely the Graph Theory, perhaps not as well-known as the other branches, but nevertheless extremely useful with important potentiality. The depiction of temporal activities and schedules through PERT diagrams (Alsina, 2011) or Activity Networks (Sommerville, 1989), as well as the depiction of structures like an organization chart or a communication network, are all expressed by using graphs.

The emergence of graphs initiated from a recreational problem that was solved by the famous Leonhard Euler (1707-1783) in 1736 (Harju, 2011). The old city of Königsberg was crossed by a river (Pregel) and its tributaries that divided the city in four sectors (A, B, C, D). Seven bridges connected the four sectors (Fig. 1a). The problem was how to find a path, visiting the four sectors, that each bridge would be crossed only once. By using a graph (Fig. 1b), Euler proved that the problem does not have a solution. In the graph, each sector was depicted by a bubble and each bridge by a line connecting two bubbles.

Since then a lot have happened. The American Frank Harary (1921-2005) is regarded as the “father” of modern Graph Theory (Alsina, 2011). He applied this theory, apart from Mathematics, to Computing, Anthropology, Linguistics, Geography, Physics, Engineering, Operational Research, Arts, Music, etc. (Harary, 1969). Other eminent scholars, like W.T. Tutte (1917-2002), E.W. Dijkstra (1930-2002) and P. Erdös (1913-1996) had a most significant contribution to Graph Theory.

2. Graph Theory

A graph is a simple and understandable way of depicting connections, relations, and interactions between all sorts of entities. The entities are depicted by a bubble, which is called “node”. The relations amongst nodes are depicted by lines or arrows, which are called “edges” (Ruohonen, 2013). Both nodes and edges can be labeled. The label usually denotes a name/identity for the node and an attribute for the edge. Thus, a graph is a set of nodes and edges (labeled or not). Whenever the direction of the connection between nodes is meaningful, the edge is depicted by an arrow and the graph is called a “directed” one or “bigraph”. The attribute of an edge is called “weight”. The number of edges that converge to a single node are called the “degree” of the node. A “loop” is an edge that starts and finishes to the same node. A “path” is a set of consecutive edges. A path is called “critical” when the sum of the weights of its edges is the minimum or the maximum, depending on the problem, compared to all the other paths of the graph. Such a graph is called “weighted”. A graph without loops is called “acyclic”. A graph that has only one edge connecting two whichever nodes is called a “tree”. These are the essential definitions of Graph Theory for the purpose of this article.

Some well-known examples of graphs, used in Public Administration, are the following:

- An organization chart is a tree(-graph). The nodes (i.e., Departments) are labeled. The edges are the lines connecting the successive levels of the organization. Although the edges are not directed, the direction is implied when for example the chain of command is perceived.

- A PERT diagram is a labeled and weighted directed graph. The critical path of such a graph is the shortest temporal one. Therefore, the weight denotes time-units (i.e., days, weeks, months or years).

Because of their versatility, graphs can be used for describing problems of Public Administration and for suggesting solutions that can be proved with mathematical rigour. Their usage is also applied to systemic theory techniques, like OMAS-III (Papakitsos, 2013a), originating from computer science techniques for problem’s analysis (Papakitsos, 2013b). In this particular method, the study of a problem / phenomenon is conducted via the application of the fundamental questions (“Journalist questions”), depicted accordingly through graphs (Fig. 2). The process is fully compatible to military decision-making (Dimitropoulos, 2013). Two relevant case-studies will be exemplified concerning the chain of command and the taxation policy.

3. Chain of Command

The chain of command is a typical feature of every hierarchical organization, like a public agency. It is depicted by a tree-graph (Fig. 3a), where the data-flow is bidirectional: information and reports are transmitted bottom-up, while directives and supervision are transmitted top-down (GPAB, 1995: 26-27).
Every level of nodes is an administration echelon. Every intermediate node, except from the top-one (“root”) or the bottom ones (“leaves”), is a dual-transmission point. The architecture is equivalent to a typical communication network. If a transmitter is corrupted then the signal becomes corrupted too, in both directions.

Fig. 2: The Basic Block Diagram of OMAS-III.

Such a corruption of transmission may happen in a typical hierarchical organization, during and immediately after the time of promotions of leaders, which usually happen in the same period and gradually for all the levels of a public agency. Leaders of the lower nodes (levels) are promoted to the next (higher) nodes and they are assigned new duties. After the assignment, they require an “adaptation period”, during which they have to be accustomed to the new environment (i.e., nature of duties and features of personnel). Yet, it is also usual that such an adaptation period is a luxury when crucial decisions have to be made. The new leader is still not well-informed about the abilities of his/her department and activities go off behind schedules. The author has recorded typical delays of a couple of months, at least, in such occasions. The same situation is observed when a leader leaves his/her post suddenly, for whatever reason. Unlike other cases of principles for public management practice (Saidin & Badara, 2014; SIGMA, 2014; Ugoani, 2014; Jahrami, 2012; Samiah & Adelabu, 2012; Vasista, 2012; Bresser-Pereira, 2005; Feldman & Khademian, 2001), the principles of “consistency” and “continuation” are rarely addressed. Without the continuation of management, the consistency of public services cannot be easily achieved.

The practice of confronting the continuation of command is the appointment of a deputy-leader. The shortcoming of the deputy’s status, which is frequently observed by the author, is that it is temporary, or at least everyone thinks so. None important decision is made by a deputy, unless the situation is extreme, although the deputy is well-informed and aware of the current conditions, better than anyone else at that time. The previous description is depicted by the tree-graph of Fig. 3b, while the previous situation is described via the terminology of OMAS-III. The transmission path, from a child-node (lower level) to the parent-node (upper-level) through the leader-node, has a weight of {4}, while the path through the deputy-node has a weight of {2}, denoting the lesser temporary status. Hence, the encountered phenomenon (“What”) is the promotion practice, conducted in such a manner that will ensure the continuity and the increased status of command (“Why”). The conditions (“How”) are described by the relevant regulations of the particular public agency for all the participants (“Who”). The input data concerns the involved leaders, while the structure of the promotion process is described by the spatial and temporal aspects (“When” & “Where”) of this situation through the application of graphs (Fig. 3).
4. Taxation Policy

It is trivial to say that every single activity of Public Governance is eventually depended on taxation. The policy on both the direct and the indirect taxation is crucial for the normal functionality of a state. The usual practices are problematic in both cases, as it will be exemplified with the case-study of the Greek taxation policy, which is by no means unique. According to the previous methodology (see OMAS-III in the Chain of Command), the encountered problem (“What”) is the designing of a taxation policy that will be efficient and effective (“Why”). The conditions (“How”) are described by the current policies (direct-indirect taxes) that are applied to all the different categories of tax-payers (“Who”). The input data concerns the expected versus the observed amount of governmental income through taxation, while the spatial and temporal aspects (“Where” & “When”) define once again the structure of the problem (Fig. 4 and Fig. 5).

Starting with the direct taxation, every adult working citizen has to contribute his/her share of taxes. In some states, even an unemployed person that owns some estate of no income must submit the annual “tax-declaration”. Corporations (enterprises, organizations, etc) are also obligated to submit a tax-declaration, which in any case records the annual income that tax is based on. The individual tax-payers can be classified into four categories: employers, employees, self-employed and unemployed estate owners. These amounted to 5,800,000, according to the relevant national Greek agency for the fiscal year 2011 (GSIS, 2011). The respective number of corporations amounted to 200,000 for the same fiscal year (2011). This is a total of 6,000,000 declarations which is a huge burden for the local tax-collecting agencies of the country, considering as well that the number of the total of civil servants has been decreased by 1/3 (approx. 300,000 less) from 2010 to 2014. The situation is depicted in Fig. 4a by a bigraph, denoting the flow of income.
Corporations (C) contribute taxes to the state (S) and salaries or shares to employers (e1) and employees (e2). The latter (e1, e2) contribute, as well, taxes to the state, along with the self-employed persons (e3) and the estate-owners (e4). The edges denote declarations of income too. The degree of node “S” is huge. The lowest “C” node denotes some not-for-profit organizations. Yet, the source of income is the “C” nodes, which attribute taxes both on their behalf and on behalf of employees (e2) and employers (e1). By relieving the nodes “e1” and “e2” from submitting declarations of income (Fig. 4b), which is rather pointless, the burden is significantly lightened (by a few millions of declarations, which is a lot of paper work). The same notion could be applied with self-employed persons. These are professionals who are registered in Chambers, Unions and similar legal entities, having an annual subscription. This subscription could be extended to cover tax contribution as well, which can be equivalent to the status of the employee.

Fig. 4: Graphs for the flow of Income

Namely, it is as if the self-employed person is an employee of the relevant Chamber/Union (Fig. 4c). Finally, the unemployed estate-owners need not to declare anything, unless they change the status of their property by buying or selling. Thus, tax-collecting agencies will have to process just 5% of the original load of work (200,000 cases). Any potential losses of tax will be compensated for exerting better control on the tax-collection process and less expenses for the public agency.

The most distinct case of indirect taxation is VAT. Judging from the numbers, VAT is almost a total failure. According to a report of the European Commission for Greece (EC, 2014), regarding the fiscal year 2011, 1/3 of the potentially collected VAT is not attributed to the state. For Greece this was amounted to 6.6 billion euros (2011) and the situation is not much better in many other countries of the European Union. Since VAT is paid by consumers (C_i) and attributed by enterprises (E), the expected situation is depicted in Fig. 5a and the observed one in Fig. 5b.
It is obvious that node “E” (Fig. 5) is corrupted, while the massive scale of this phenomenon makes it not amenable to patching up. There are two affordable solutions for the state: either abandoning the notion of VAT altogether in favour of an increase to direct income-taxes (Khan & Khan, 2014) or perhaps pre-collecting VAT from enterprises based on their (inflow) procurements. The alternative of monitoring the transactions electronically (OECD, 2014) is not applicable to every society or region (i.e., through credit cards), while in any case “the simpler the better”. Systemically, the most stable systems are the simplest ones.

5. Conclusions

In the present paper, an overview of the graph-theory has been presented, stressing its essential concepts and definitions. As a branch of Discrete Mathematics, it is claimed that graphs can be a valuable tool for decision making, by providing a simplified picture of various phenomena and situations. The value of graphs has been demonstrated by describing two significant cases of Public Governance interest: the Chain of Command and the taxation policy. In both cases, the nature of the situations, some encountered problems and some suggested solutions were depicted and analyzed through the usage of graphs, in a systemic manner.

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